

Exercise 53

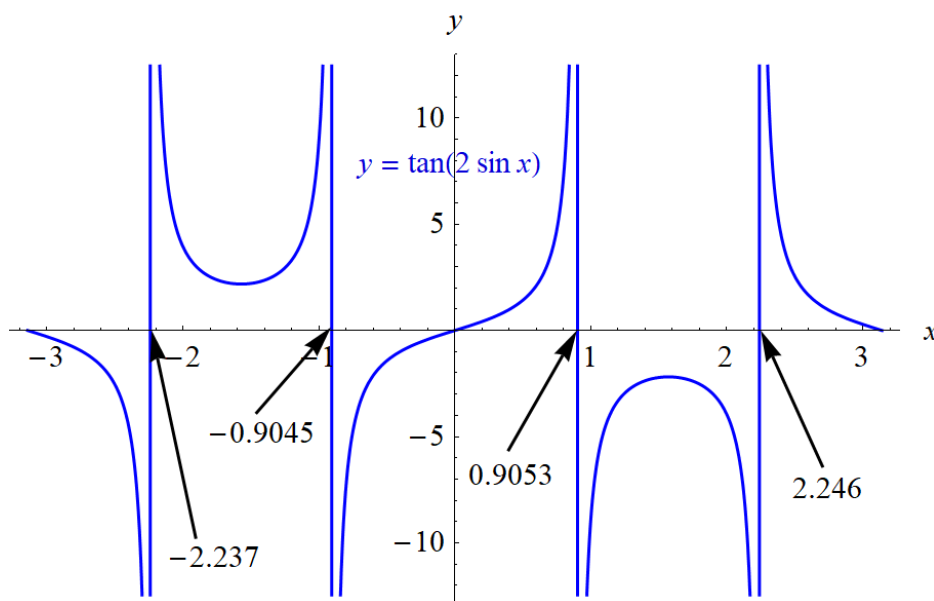
Use a graph to estimate the equations of all the vertical asymptotes of the curve

$$y = \tan(2 \sin x) \quad -\pi \leq x \leq \pi$$

Then find the exact equations of these asymptotes.

Solution

Below is a graph of $y(x)$ versus x over $-\pi \leq x \leq \pi$.



The estimated vertical asymptotes over this interval are $x = -2.237$ and $x = -0.9045$ and $x = 0.9053$ and $x = 2.246$. To find them exactly, rewrite the function.

$$y = \tan(2 \sin x) = \frac{\sin(2 \sin x)}{\cos(2 \sin x)}$$

The vertical asymptotes occur where the denominator is zero.

$$\cos(2 \sin x) = 0$$

$$2 \sin x = \frac{1}{2}(2n - 1)\pi, \quad n = 0, \pm 1, \pm 2, \dots$$

$$\sin x = \frac{\pi}{4}(2n - 1)$$

Since the sine has to be 1 or smaller, only $n = 0$ and $n = 1$ are allowed.

$$\sin x = -\frac{\pi}{4} \quad \Rightarrow \quad x = \left\{ -\sin^{-1}\left(\frac{\pi}{4}\right), \sin^{-1}\left(\frac{\pi}{4}\right) - \pi \right\} \approx \{-0.903339, -2.23825\}$$

$$\sin x = \frac{\pi}{4} \quad \Rightarrow \quad x = \left\{ \sin^{-1}\left(\frac{\pi}{4}\right), \pi - \sin^{-1}\left(\frac{\pi}{4}\right) \right\} \approx \{0.903339, 2.23825\}$$